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Reparameterization of the scalar potential in the two Higgs doublet model

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Abstract. In the two Higgs doublet model with no additional symmetries in the scalar sector (different from the gauge and Lorentz symmetries), it is customary to reparameterize the model by rotating the scalar doublets so that one of the vacuum expectation values vanishes. It is well known that the Yukawa sector of the model is unaffected by such a transformation. Notwithstanding this, since the Higgs potential must also be transformed, it is necessary to show that such a sector is also unaltered in its physical content. We demonstrate that the physical content of the potential is invariant even when the charge conjugation symmetry is demanded.

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1 Introduction

One of the simplest extension of the SM is the so called two Higgs doublet model (2HDM), which consists of adding a new doublet with the same quantum numbers as the former. The most studied of this kind of models are the so called 2HDM type I and 2HDM type II, according to the discrete symmetry imposed in the scalar sector that provides different Yukawa couplings in the up and down sectors. In the model of type I, only one Higgs doublet gives mass to the up and down sectors; while in the model of type II one doublet gives mass to the up-type quarks while the other doublet provides the mass to the down-type quarks. Since each doublet could acquire a vacuum expectation value (VEV), the presence of two different VEV's enables us to explain the hierarchy mass problem in the third generation of quarks. In the case in which no discrete symmetry is introduced the two doublets can couple to both types of quarks, generating flavor changing neutral currents (FCNC); because in this case the mass matrix is generated by two different matrices which cannot be diagonalized at the same time. In this case we talk of 2HDM type III. In the case of the 2HDM III, it is possible to make a rotation between the two Higgs doublets which is equivalent to choose a new basis, and the physical content of the Lagrangian remains invariant. This fact permits one to make a rotation so that one of the VEV's vanishes.

In general the 2HDM has two CP-even neutral scalar Higgs bosons (h^0, H^0) which are mixed through an angle denoted as α; two CP-odd scalar fields (A^0, G_Z^0) where the first correspond to a physical particle and the second is a Goldstone boson associated with Z_μ; and finally, two types of charged fields $H^\pm, G_W^\pm$ which correspond to two scalar charged particles and two would be Goldstone bosons associated with $W^\pm$. They are mixed through an angle denoted by β where $\tan\beta = v_2/v_1$ with v_1 v_2 denoting the two VEV's. However, in the model where one of the VEV is taken out by the rotation, the would be Goldstone bosons and scalar bosons do not mix because they belong to different doublets due to the redefinition of the coordinate system for the doublets. For the gauge group $SU(N)$ with the m multiplet fundamental representation which gets the VEV $\langle H_i \rangle = v_i$, it is always possible to rotate the system so that $\langle H_1 \rangle = v_1$ and $\langle H_i \rangle = 0$ with $i = 2, \ldots, m$ [1]. Recent studies of the invariance of the physical parameters under such a rotation have been carried out. For instance [3] discusses the independence of the physical parameters from basis changes in the Lagrangian, in a more observable oriented manner with an eye on the minimal supersymmetric standard model. On the other hand, a more theoretical approach can be seen in [2, 4]; in [2] the most general 2HDM is rewritten in an explicitly $SU(2)$-covariant form, while in [4] the invariance is presented based on group theoretical grounds, viewing the two Higgs doublets as components of a generic "hyperspinor".

In summary, in the absence of additional symmetries (different from the gauge and Lorentz symmetry) in the scalar sector, we can always choose a basis in which only one doublet acquires a vacuum expectation value (VEV) without affecting the physical meaning of the model [1]. This change of basis consists of a unitary transformation between the two Higgs doublets that cancels one of the

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VEV's. Nevertheless, the literature usually studies this rotation in the Yukawa sector only. However, even in the case in which no additional symmetry is taken in the Yukawa sector, charge conjugation invariance (C-invariance) is assumed in the Higgs potential [5, 6], and since the same choice of basis must be done in this sector, it is necessary to check that a change of basis keeps the Higgs potential unaltered in its physical content. The purpose of the present paper is to show explicitly that the potential maintains its physical meaning even when C-invariance is imposed as long as no additional symmetries in the potential are demanded.

2 Rotation of the Yukawa Lagrangian in the 2HDM type III

We start defining two Higgs doublets with VEV's:

$$\Phi'_{1,2} = \begin{pmatrix} (\phi^+_{1,2})' \\ (\phi^0_{1,2})' \end{pmatrix} = \begin{pmatrix} (\phi^+_{1,2})' \\ (h_1 + v_1 + \mathrm{i}g_1)' \end{pmatrix} ; \quad \widetilde{\Phi}'_{1,2} = \mathrm{i}\sigma_2 \Phi'_{1,2} \, ; \; \langle \Phi'_{1,2} \rangle = v'_{1,2} \tag{1}$$

and writing the Yukawa Lagrangian of the most general two Higgs doublet model (the so called 2HDM type III)

$$-\mathcal{L}_{\mathrm{Y}} = \widetilde{\eta}^{U,0}_{ij} \overline{Q}^0_{i\mathrm{L}} \widetilde{\Phi}'_1 U^0_{j\mathrm{R}} + \widetilde{\eta}^{D,0}_{ij} \overline{Q}^0_{i\mathrm{L}} \Phi'_1 D^0_{j\mathrm{R}} + \widetilde{\xi}^{U,0}_{ij} \overline{Q}^0_{i\mathrm{L}} \widetilde{\Phi}'_2 U^0_{j\mathrm{R}} + \widetilde{\xi}^{D,0}_{ij} \overline{Q}^0_{i\mathrm{L}} \Phi'_2 D^0_{j\mathrm{R}} + \text{lepton sector} + \text{h.c.} \, , \tag{2}$$

where $Q^0_{i\mathrm{L}}$ denotes the left-handed quark doublets with i the family index, and $U^0_{j\mathrm{R}}$, $D^0_{j\mathrm{R}}$ correspond to the right-handed singlets of up-type and down-type quarks respectively. The superscript "0" means that we are dealing with gauge eigenstates. Finally, $\widetilde{\eta}^{U,0}_{ij}$, $\widetilde{\xi}^{D,0}_{ij}$ correspond to the Yukawa vertices giving in general a mixing among families. We can make a rotation between the doublets,

$$\begin{pmatrix} \Phi_1 \\ \Phi_2 \end{pmatrix} \equiv \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \Phi'_1 \\ \Phi'_2 \end{pmatrix} \tag{3}$$

and define some new rotated Yukawa couplings,

$$\begin{pmatrix} \eta^{(U,D),0}_{ij} \\ \xi^{(U,D),0}_{ij} \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \widetilde{\eta}^{(U,D),0}_{ij} \\ \widetilde{\xi}^{(U,D),0}_{ij} \end{pmatrix} . \tag{4}$$

We shall deal with the quark sector only since the results for the lepton sector will be straightforward. In terms of Φ_1, Φ_2, $\eta^{(U,D),0}_{ij}$ and $\xi^{(U,D),0}_{ij}$ the Yukawa Lagrangian could be rewritten as

$$-\mathcal{L}_{\mathrm{Y}} = \overline{Q}^0_{i\mathrm{L}} \eta^{U,0}_{ij} \widetilde{\Phi}_1 U^0_{j\mathrm{R}} + \overline{Q}^0_{i\mathrm{L}} \eta^{D,0}_{ij} \Phi_1 D^0_{j\mathrm{R}} + \overline{Q}^0_{i\mathrm{L}} \xi^{U,0}_{ij} \widetilde{\Phi}_2 U^0_{j\mathrm{R}} + \overline{Q}^0_{i\mathrm{L}} \xi^{D,0}_{ij} \Phi_2 D^0_{j\mathrm{R}} + \text{h.c.} \, ,$$

with the same form as the original Lagrangian if we forget the prime notation. Consequently, the combined rotations (3) and (4) do not have physical consequences since they are basically a change of basis. In particular we can choose $\theta = \beta$ where $\tan\beta \equiv v'_2/v'_1$ so that

$$\langle \Phi_1 \rangle = \cos\beta \langle \Phi'_1 \rangle + \sin\beta \langle \Phi'_2 \rangle = \sqrt{v'^2_1 + v'^2_2} \equiv v \, , \quad \langle \Phi_2 \rangle = -\sin\beta \langle \Phi'_1 \rangle + \cos\beta \langle \Phi'_2 \rangle = 0 \, . \tag{5}$$

In this case we managed to get $\langle \Phi_2 \rangle = 0$. Since this Lagrangian contains exactly the same physical information as the first one, we conclude that in model type III the parameter $\tan\beta$ is totally spurious, and we can assume without any loss of generality that one of the VEV's is zero.

On the other hand, it is possible to reverse the steps above and start from the representation in which $\langle \Phi_2 \rangle = 0$ (the "fundamental representation") and make a rotation of the Higgs doublets from which the $\tan\beta$ parameter arises. Although these rotations provide a spurious parameter ($\tan\beta$), they have the advantage of making the comparison between the model type II with the model type III more apparent [7], and the same for the comparison between model type I and type III. It is important to emphasize that the mixing matrices $\eta^{(U,D),0}_{ij}$, $\xi^{(U,D),0}_{ij}$ depend explicitly on the $\tan\beta$ parameter; thus, they are basis dependent. Nevertheless, it can be explicitly shown that the whole of the couplings are basis independent [8] as expected.

We should emphasize that the invariance of the Lagrangian (2) under the rotation (3) requires no additional symmetries to be imposed in such a Lagrangian. Notwithstanding this, even when the Lagrangian (2) is left in the most general form it is customary to impose a C-invariance in the Higgs potential, and since the rotation (3) must be applied to such a potential as well, we should check that the transformation described by (3) still leaves the potential unaltered in its physical content.

3 Rotation in the Higgs potential

After examining the rotation in the Yukawa sector, we shall see how this transformation changes the parameters in the potential, and show that the physical content of the potential remains intact. For this we should examine the physical parameters of the model and show that they are basis invariant.

3.1 Transformation of the parameters in the potential

Let us start from an arbitrary parameterization in which both VEV's are in general different from zero. The most general renormalizable and gauge invariant potential reads

$$V_g = -\widetilde{\mu}^2_1 \widehat{A}' - \widetilde{\mu}^2_2 \widehat{B}' - \widetilde{\mu}^2_3 \widehat{C}' - \widetilde{\mu}^2_4 \widehat{D}' + \widetilde{\lambda}_1 \widehat{A}'^2 + \widetilde{\lambda}_2 \widehat{B}'^2 + \widetilde{\lambda}_3 \widehat{C}'^2 + \widetilde{\lambda}_4 \widehat{D}'^2 + \widetilde{\lambda}_5 \widehat{A}' \widehat{B}' + \widetilde{\lambda}_6 \widehat{A}' \widehat{C}' + \widetilde{\lambda}_8 \widehat{A}' \widehat{D}' + \widetilde{\lambda}_7 \widehat{B}' \widehat{C}' + \widetilde{\lambda}_9 \widehat{B}' \widehat{D}' + \widetilde{\lambda}_{10} \widehat{C}' \widehat{D}' \, , \tag{6}$$

where we have defined the four independent gauge invariant hermitian operators

$$\widehat{A}' \equiv \Phi'^\dagger_1 \Phi'_1 \, , \quad \widehat{B}' \equiv \Phi'^\dagger_2 \Phi'_2 , \quad \widehat{C}' \equiv \frac{1}{2} \left(\Phi'^\dagger_1 \Phi'_2 + \Phi'^\dagger_2 \Phi'_1 \right) = \mathrm{Re} \left(\Phi'^\dagger_1 \Phi'_2 \right) , \quad \widehat{D}' \equiv -\frac{i}{2} \left(\Phi'^\dagger_1 \Phi'_2 - \Phi'^\dagger_2 \Phi'_1 \right) = \mathrm{Im} \left(\Phi'^\dagger_1 \Phi'_2 \right) . \tag{7}$$

In Sect. 2 we showed that the rotation described by (3) can be done without changing the physical content of the Yukawa Lagrangian. In order to show the invariance of the physical content in the scalar potential, we shall calculate the way in which the parameters $\widetilde{\mu}_i$, $\widetilde{\lambda}_i$ transform under this rotation. First, we calculate the way in which the operators $\widehat{A}'$, $\widehat{B}'$, $\widehat{C}'$, $\widehat{D}'$ transform. Taking into account (3) we get

$$\begin{aligned}\widehat{A}' &\equiv \Phi_1'^{\dagger}\Phi_1' = \left(\Phi_1^{\dagger}\cos\theta - \Phi_2^{\dagger}\sin\theta\right)(\Phi_1\cos\theta - \Phi_2\sin\theta) \\ &= \Phi_1^{\dagger}\Phi_1\cos^2\theta - 2\cos\theta\sin\theta\left(\frac{\Phi_1^{\dagger}\Phi_2 + \Phi_2^{\dagger}\Phi_1}{2}\right) \\ &\quad + \Phi_2^{\dagger}\Phi_2\sin^2\theta \\ &= \widehat{A}\cos^2\theta + \widehat{B}\sin^2\theta - \sin 2\theta\widehat{C}\,,\end{aligned}$$

and we obtain the transformations for the other operators in a similar way, the results reading

$$\begin{aligned}\widehat{A}' &= \widehat{A}\cos^2\theta + \widehat{B}\sin^2\theta - \widehat{C}\sin 2\theta\,, \\ \widehat{B}' &= \widehat{A}\sin^2\theta + \widehat{B}\cos^2\theta + \widehat{C}\sin 2\theta\,, \\ \widehat{C}' &= \frac{1}{2}\widehat{A}\sin 2\theta - \frac{1}{2}\widehat{B}\sin 2\theta + \widehat{C}\cos 2\theta\,, \\ \widehat{D}' &= \widehat{D}\,, \\ \widehat{A}'^2 &= \widehat{A}^2\cos^4\theta + \widehat{B}^2\sin^4\theta + \widehat{C}^2\sin^2 2\theta + \frac{1}{2}\widehat{A}\widehat{B}\sin^2 2\theta \\ &\quad - 2\widehat{A}\widehat{C}\sin 2\theta\cos^2\theta - 2\widehat{B}\widehat{C}\sin^2\theta\sin 2\theta\,, \\ \widehat{B}'^2 &= \widehat{A}^2\sin^4\theta + \widehat{B}^2\cos^4\theta + \widehat{C}^2\sin^2 2\theta + \frac{1}{2}\widehat{A}\widehat{B}\sin^2 2\theta \\ &\quad + 2\widehat{A}\widehat{C}\sin 2\theta\sin^2\theta + 2\widehat{B}\widehat{C}\sin 2\theta\cos^2\theta\,, \\ \widehat{C}'^2 &= \frac{1}{4}\left(\widehat{A}^2 + \widehat{B}^2\right)\sin^2 2\theta + \widehat{C}^2\cos^2 2\theta - \frac{1}{2}\widehat{A}\widehat{B}\sin^2 2\theta \\ &\quad + \frac{1}{2}\widehat{A}\widehat{C}\sin 4\theta - \frac{1}{2}\widehat{B}\widehat{C}\sin 4\theta\,, \\ \widehat{D}'^2 &= \widehat{D}^2\,, \\ \widehat{A}'\widehat{B}' &= \left(\frac{1}{4}\widehat{A}^2 + \frac{1}{4}\widehat{B}^2 - \widehat{C}^2\right)\sin^2 2\theta + \widehat{A}\widehat{B}\left(\cos^4\theta + \sin^4\theta\right) \\ &\quad + \left(\widehat{A}\widehat{C} - \widehat{B}\widehat{C}\right)\sin 2\theta\cos 2\theta\,, \\ \widehat{A}'\widehat{C}' &= \frac{1}{2}\widehat{A}^2\sin 2\theta\cos^2\theta - \frac{1}{2}\widehat{B}^2\sin^2\theta\sin 2\theta \\ &\quad - \widehat{C}^2\sin 2\theta\cos 2\theta - \frac{1}{4}\widehat{A}\widehat{B}\sin 4\theta \\ &\quad + \widehat{A}\widehat{C}\left(4\cos^2\theta - 3\right)\cos^2\theta \\ &\quad + \widehat{B}\widehat{C}\left(4\cos^2\theta - 1\right)\sin^2\theta\,, \\ \widehat{A}'\widehat{D}' &= \widehat{A}\widehat{D}\cos^2\theta + \widehat{B}\widehat{D}\sin^2\theta - \widehat{C}\widehat{D}\sin 2\theta\,, \\ \widehat{B}'\widehat{C}' &= \frac{1}{2}\widehat{A}^2\sin 2\theta\sin^2\theta - \frac{1}{2}\widehat{B}^2\sin 2\theta\cos^2\theta + \frac{1}{2}\widehat{C}^2\sin 4\theta \\ &\quad + \frac{1}{4}\widehat{A}\widehat{B}\sin 4\theta + \widehat{A}\widehat{C}\left(\cos 2\theta + 2\cos^2\theta\right)\sin^2\theta \\ &\quad + \widehat{B}\widehat{C}\left(\cos 2\theta - 2\sin^2\theta\right)\cos^2\theta\,, \\ \widehat{B}'\widehat{D}' &= \widehat{A}\widehat{D}\sin^2\theta + \widehat{B}\widehat{D}\cos^2\theta + \widehat{C}\widehat{D}\sin 2\theta\,, \\ \widehat{C}'\widehat{D}' &= \frac{1}{2}\left(\widehat{A}\widehat{D} - \widehat{B}\widehat{D}\right)\sin 2\theta + \widehat{C}\widehat{D}\cos 2\theta\,. \end{aligned} \tag{8}$$

Now, we can build up a new parameterization of the potential so that

$$\begin{aligned}V_g &= -\mu_1^2\widehat{A} - \mu_2^2\widehat{B} - \mu_3^2\widehat{C} - \mu_4^2\widehat{D} + \lambda_1\widehat{A}^2 + \lambda_2\widehat{B}^2 + \lambda_3\widehat{C}^2 \\ &\quad + \lambda_4\widehat{D}^2 + \lambda_5\widehat{A}\widehat{B} + \lambda_6\widehat{A}\widehat{C} + \lambda_8\widehat{A}\widehat{D} + \lambda_7\widehat{B}\widehat{C} \\ &\quad + \lambda_9\widehat{B}\widehat{D} + \lambda_{10}\widehat{C}\widehat{D}\,;\end{aligned} \tag{9}$$

in order to find the values of μ_i, λ_i in terms of $\widetilde{\mu}_i$, $\widetilde{\lambda}_i$, we use (6) and (8) to write e.g. the coefficient proportional to the operator $\widehat{A}$. These terms are compared with the term proportional to the operator $\widehat{A}$ in (9) obtaining

$$-\mu_1^2\widehat{A} = \left(-\widetilde{\mu}_1^2\cos^2\theta - \widetilde{\mu}_2^2\sin^2\theta - \widetilde{\mu}_3^2\sin\theta\cos\theta\right)\widehat{A}\,;$$

therefore, the coefficient μ_1^2 is related to the parameters $\widetilde{\mu}_i$, $\widetilde{\lambda}_i$ in the following way:

$$\mu_1^2 = \left(\widetilde{\mu}_1^2\cos^2\theta + \widetilde{\mu}_2^2\sin^2\theta + \frac{1}{2}\widetilde{\mu}_3^2\sin 2\theta\right)$$

by the same token, the other sets of the parameters μ_i, λ_i are related to the $\widetilde{\mu}_i$, $\widetilde{\lambda}_i$ parameters in the following way:

$$\begin{aligned}\mu_1^2 &= \left(\widetilde{\mu}_1^2\cos^2\theta + \widetilde{\mu}_2^2\sin^2\theta + \frac{1}{2}\widetilde{\mu}_3^2\sin 2\theta\right)\,, \\ \mu_2^2 &= \left(\widetilde{\mu}_1^2\sin^2\theta + \widetilde{\mu}_2^2\cos^2\theta - \frac{1}{2}\widetilde{\mu}_3^2\sin 2\theta\right)\,, \\ \mu_3^2 &= \left(-\widetilde{\mu}_1^2\sin 2\theta + \widetilde{\mu}_2^2\sin 2\theta + \widetilde{\mu}_3^2\cos 2\theta\right)\,, \quad \mu_4^2 = \widetilde{\mu}_4^2\,, \\ \lambda_1 &= \widetilde{\lambda}_1\cos^4\theta + \widetilde{\lambda}_2\sin^4\theta + \frac{1}{4}\left(\widetilde{\lambda}_3 + \widetilde{\lambda}_5\right)\sin^2 2\theta \\ &\quad + \frac{1}{2}\left(\widetilde{\lambda}_6\cos^2\theta + \widetilde{\lambda}_7\sin^2\theta\right)\sin 2\theta\,, \\ \lambda_2 &= \widetilde{\lambda}_1\sin^4\theta + \widetilde{\lambda}_2\cos^4\theta + \frac{1}{4}\left(\widetilde{\lambda}_3 + \widetilde{\lambda}_5\right)\sin^2 2\theta \\ &\quad - \frac{1}{2}\left(\widetilde{\lambda}_6\sin^2\theta + \widetilde{\lambda}_7\cos^2\theta\right)\sin 2\theta\,, \\ \lambda_3 &= \left(\widetilde{\lambda}_1 + \widetilde{\lambda}_2 - \widetilde{\lambda}_5\right)\sin^2 2\theta + \widetilde{\lambda}_3\cos^2 2\theta \\ &\quad + \frac{1}{2}\left(\widetilde{\lambda}_7 - \widetilde{\lambda}_6\right)\sin 4\theta\,, \\ \lambda_4 &= \widetilde{\lambda}_4\,, \\ \lambda_5 &= \frac{1}{2}\left(\widetilde{\lambda}_1 + \widetilde{\lambda}_2 - \widetilde{\lambda}_3\right)\sin^2 2\theta + \widetilde{\lambda}_5\left(\cos^4\theta + \sin^4\theta\right) \\ &\quad + \frac{1}{4}\left(\widetilde{\lambda}_7 - \widetilde{\lambda}_6\right)\sin 4\theta\,, \\ \lambda_6 &= 2\left(\widetilde{\lambda}_2\sin^2\theta - \widetilde{\lambda}_1\cos^2\theta\right)\sin 2\theta + \frac{1}{2}\left(\widetilde{\lambda}_3 + \widetilde{\lambda}_5\right)\sin 4\theta \\ &\quad + \widetilde{\lambda}_6\left(4\cos^2\theta - 3\right)\cos^2\theta + \widetilde{\lambda}_7\left(\cos 2\theta + 2\cos^2\theta\right)\sin^2\theta\,, \\ \lambda_7 &= 2\left(\widetilde{\lambda}_2\cos^2\theta - \widetilde{\lambda}_1\sin^2\theta\right)\sin 2\theta - \frac{1}{2}\left(\widetilde{\lambda}_3 + \widetilde{\lambda}_5\right)\sin 4\theta \\ &\quad + \widetilde{\lambda}_6\left(4\cos^2\theta - 1\right)\sin^2\theta + \widetilde{\lambda}_7\left(\cos 2\theta - 2\sin^2\theta\right)\cos^2\theta\,, \\ \lambda_8 &= \widetilde{\lambda}_8\cos^2\theta + \widetilde{\lambda}_9\sin^2\theta + \frac{1}{2}\widetilde{\lambda}_{10}\sin 2\theta\,, \\ \lambda_9 &= \left(\widetilde{\lambda}_8\sin^2\theta + \widetilde{\lambda}_9\cos^2\theta - \frac{1}{2}\widetilde{\lambda}_{10}\sin 2\theta\right)\,, \\ \lambda_{10} &= \left[\left(\widetilde{\lambda}_9 - \widetilde{\lambda}_8\right)\sin 2\theta + \widetilde{\lambda}_{10}\cos 2\theta\right]\,.\end{aligned} \tag{10}$$

3.2 Tadpoles

From now on, we shall consider the potential with invariance under charge conjugation [6]. Under this transformation, a Higgs doublet Φ_i of hypercharge 1 transforms as $\Phi_i \to e^{i\alpha_i}\Phi_i^*$ where the parameters α_i are arbitrary [6]. Consequently, under charge conjugation we obtain $\Phi_i^\dagger\Phi_j \to e^{i(\alpha_j-\alpha_i)}\Phi_j^\dagger\Phi_i$. In particular, if we choose $\alpha_i = \alpha_j$ the operator $\widehat{D}$ defined by (9) and (7) reverse sign under C-conjugation, while the other operators are invariant[1]. Therefore, imposition of C-invariance leads to $\mu_4 = \lambda_8 = \lambda_9 = \lambda_{10} = 0$ in the potential (9). However, no other discrete or continuous symmetry is assumed. In that case the tadpoles are given by

$$T = T_3 h_1 + T_7 h_2$$

where

$$\begin{aligned} T_3 \equiv & -\mu_1^2 v_1 - \frac{1}{2}\mu_3^2 v_2 + \lambda_1 v_1^3 + \frac{1}{2}\lambda_3 v_1 v_2^2 + \frac{1}{2}\lambda_5 v_1 v_2^2 \\ & + \frac{3}{4}\lambda_6 v_1^2 v_2 + \frac{1}{4}\lambda_7 v_2^3 , \\ T_7 \equiv & -\mu_2^2 v_2 - \frac{1}{2}\mu_3^2 v_1 + \lambda_2 v_2^3 + \frac{1}{2}\lambda_3 v_1^2 v_2 + \frac{1}{2}\lambda_5 v_1^2 v_2 \\ & + \frac{1}{4}\lambda_6 v_1^3 + \frac{3}{4}\lambda_7 v_2^2 v_1 , \end{aligned} \tag{11}$$

and these tadpoles coincide with the minimum conditions, applying $\mu_4 = \lambda_8 = \lambda_9 = \lambda_{10} = 0$. Now, we find the relation among the tadpoles in both parameterizations by using (11), and (1) and (3). We have

$$\begin{aligned} T_3 h_1 + T_7 h_2 &= T_3 \left(h_1' \cos\theta + h_2' \sin\theta\right) \\ &\quad + T_7 \left(-h_1' \sin\theta + \cos\theta h_2'\right) \\ &= \left(T_3 \cos\theta - T_7 \sin\theta\right) h_1' \\ &\quad + \left(T_3 \sin\theta + T_7 \cos\theta\right) h_2' \\ &= T_3' h_1' + T_7' h_2' , \end{aligned}$$

from which we see that the tadpoles in both parameterizations are related through the rotation

$$\begin{pmatrix} T_3' \\ T_7' \end{pmatrix} \equiv \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} T_3 \\ T_7 \end{pmatrix} . \tag{12}$$

As a proof of consistency, we can check that from (11), and from (10)) the following relations are obtained after a bit of algebra:

$$\begin{aligned} T_3 \cos\theta - T_7 \sin\theta = & -\widetilde{\mu}_1^2 v_1' - \frac{1}{2}\widetilde{\mu}_3^2 v_2' + \widetilde{\lambda}_1 v_1'^3 + \frac{1}{2}\widetilde{\lambda}_3 v_1' v_2'^2 \\ & + \frac{1}{2}\widetilde{\lambda}_5 v_1' v_2'^2 + \frac{3}{4}\widetilde{\lambda}_6 v_1'^2 v_2' + \frac{1}{4}\widetilde{\lambda}_7 v_2'^3 , \end{aligned}$$

$$\begin{aligned} T_3 \sin\theta + T_7 \cos\theta = & -\widetilde{\mu}_2^2 v_2' - \frac{1}{2}\widetilde{\mu}_3^2 v_1' + \widetilde{\lambda}_2 v_2'^3 + \frac{1}{2}\widetilde{\lambda}_3 v_1'^2 v_2' \\ & + \frac{1}{2}\widetilde{\lambda}_5 v_1'^2 v_2' + \frac{1}{4}\widetilde{\lambda}_6 v_1'^3 + \frac{3}{4}\widetilde{\lambda}_7 v_2'^2 v_1' , \end{aligned} \tag{13}$$

where

$$\begin{pmatrix} v_1' \\ v_2' \end{pmatrix} \equiv \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} \tag{14}$$

relates the VEV's between both parameterizations[2]. From (12) and (13) we get

$$\begin{aligned} T_3' = & -\widetilde{\mu}_1^2 v_1' - \frac{1}{2}\widetilde{\mu}_3^2 v_2' + \widetilde{\lambda}_1 v_1'^3 + \frac{1}{2}\widetilde{\lambda}_3 v_1' v_2'^2 + \frac{1}{2}\widetilde{\lambda}_5 v_1' v_2'^2 \\ & + \frac{3}{4}\widetilde{\lambda}_6 v_1'^2 v_2' + \frac{1}{4}\widetilde{\lambda}_7 v_2'^3 , \\ T_7' = & -\widetilde{\mu}_2^2 v_2' - \frac{1}{2}\widetilde{\mu}_3^2 v_1' + \widetilde{\lambda}_2 v_2'^3 + \frac{1}{2}\widetilde{\lambda}_3 v_1'^2 v_2' + \frac{1}{2}\widetilde{\lambda}_5 v_1'^2 v_2' \\ & + \frac{1}{4}\widetilde{\lambda}_6 v_1'^3 + \frac{3}{4}\widetilde{\lambda}_7 v_2'^2 v_1' , \end{aligned} \tag{15}$$

and comparing with (11) we see that the form of the tadpole is preserved by changing the original parameters with the prime parameters.

Finally, it can be checked that when one of the VEV vanishes, its corresponding tadpole vanishes as well; this is an important requirement to preserve the renormalizability of the theory [10].

3.3 Higgs boson masses

Another important proof of consistency is to verify that both parameterizations predict the same masses for the Higgs bosons. We shall use once again the potential with C-invariance. In a general parameterization, the minimal conditions are reduced to

$$\begin{aligned} \mu_1 v_1 = & \Big(-\frac{1}{2}\mu_3 v_2 + \lambda_1 v_1^3 + \frac{1}{2}\lambda_3 v_2^2 v_1 + \frac{1}{2}\lambda_5 v_2^2 v_1 \\ & + \frac{3}{4}\lambda_6 v_1^2 v_2 + \frac{1}{4}\lambda_7 v_2^3 \Big) , \\ \mu_2 v_2 = & \Big(-\frac{1}{2}\mu_3 v_1 + \lambda_2 v_2^3 + \frac{1}{2}\lambda_3 v_1^2 v_2 + \frac{1}{2}\lambda_5 v_1^2 v_2 + \frac{1}{4}\lambda_6 v_1^3 \\ & + \frac{3}{4}\lambda_7 v_1 v_2^2 \Big) ; \end{aligned}$$

the mass matrix is obtained by using once again $\mu_4 = \lambda_8 = \lambda_9 = \lambda_{10} = 0$. Let us start with the matrix elements corresponding to the scalar Higgs bosons m_{H^0}, m_{h^0}. If we assume that both VEV's are different from zero and utilize the minimum conditions, we obtain the following mass matrix:

$$\begin{pmatrix} M_{33}^2 & M_{37}^2 \\ M_{37}^2 & M_{77}^2 \end{pmatrix} , \tag{16}$$

[1] Of course, we could have chosen $\alpha_i - \alpha_j = \pm\pi$, in which case the operator $\widehat{C} = \mathrm{Re}\left(\Phi_1^\dagger\Phi_2\right)$ is the one that violates charge conservation. Additionally, any other choice for $\alpha_i - \alpha_j$ is possible, and in general no parameter vanishes. However, taking into account that these phases must be fixed (though arbitrary), C-invariance would impose relations among the coefficients so that the number of free parameters is always the same (for instance μ_3 and μ_4 would not be independent any more).

[2] Observe that such a rotation leaves invariant the quantity $v_1^2 + v_2^2 = v_1'^2 + v_2'^2 = \frac{2m_w^2}{g^2}$, as it should.

with

$$M_{33}^2 = \frac{1}{4v_1}\left(2\mu_3^2 v_2 + 8\lambda_1 v_1^3 + 3\lambda_6 v_1^2 v_2 - \lambda_7 v_2^3\right),$$
$$M_{37}^2 = -\frac{1}{2}\mu_3^2 + \frac{3}{4}\lambda_7 v_2^2 + \frac{3}{4}\lambda_6 v_1^2 + \lambda_3 v_1 v_2 + \lambda_5 v_1 v_2,$$
$$M_{77}^2 = \frac{1}{4v_2}\left(2\mu_3^2 v_1 + 8\lambda_2 v_2^3 - \lambda_6 v_1^3 + 3\lambda_7 v_2^2 v_1\right). \quad (17)$$

For the sake of simplicity, we just show that the determinant of this matrix (i.e. the product of the squared masses), coincides for two parameterizations connected by a transformation like (3). The mass matrix in any other parameterization with both VEV's different from zero, has the same form as (16) and (17), but replacing $\mu_i^2 \to \widetilde{\mu}_i^2$, $\lambda_i \to \widetilde{\lambda}_i$. It is a matter of cumbersome algebra to demonstrate that

$$M_{33}^2 M_{77}^2 - \left(M_{37}^2\right)^2 = \widetilde{M}_{33}^2 \widetilde{M}_{77}^2 - \left(\widetilde{M}_{37}^2\right)^2 .$$

This demonstration is carried out by taking into account the relations (10) among the parameters in both bases. In a similar fashion, we can show that the eigenvalues coincide in both bases. Therefore, the Higgs boson masses are equal in both parameterizations as they must. Finally, if the angle of rotation is chosen such that one of the VEV's is zero, (e.g. $v_2 = 0$) in one of the bases, then the minimum conditions and mass matrix elements become much simpler, and the equality is easier to demonstrate.

By the same token, we can check that for the other Higgs mass matrices the determinants and eigenvalues are invariant under the transformation (3), showing that the observables are not altered by this change of basis.

4 VEV with complex phases

If we assume that one of the VEV in the prime basis (say v_2') acquires a complex phase, we can eliminate this VEV by making a complex rotation. However, even if we start with real parameters in the potential of the prime parameterization, after the complex rotation some of these parameters acquire complex values in the new parameterization. In that case the change of basis has translated the CP violation sources from the VEV to the parameters of the potential. In other words the original spontaneous CP violation has been transformed into an explicit violation of such a symmetry.

5 Conclusions

The simplest extension of the standard model is the so called two Higgs doublet model. This is a model well motivated from both theoretical and phenomenological points of view [5]. For such a model, it is customary (in the absence of additional symmetries) to make a rotation in order to get rid of one of the vacuum expectation values. The literature has discussed this rotation in the framework of the Yukawa sector. However, it is important to check that the physical content of the potential also remains invariant under the transformation made in the Yukawa sector, especially because a charge conjugation invariance in the potential is demanded even when no additional symmetries in the Yukawa sector are demanded. We show by finding the transformation in the parameters of the potential that the physical content of the potential remains invariant even when C-invariance is demanded. Such an invariance is demonstrated by examining the tadpoles and the physical spectrum of the Higgs sector, and showing their invariance under the transformation described above. In particular, it is worthwhile emphasizing that when one of the VEV's is null, the corresponding tadpole also vanishes, and this feature is essential to preserve the renormalizability of the theory.

On the other hand, we can realize that the models type I and II have a remarkable difference with respect to the model type III, since it is well known that the former two ones are highly dependent on the $\tan\beta$ parameter, while the latter is not. We can see the difference from the point of view of symmetries: the 2HDM is constructed in such a way that we make an exact "duplicate" of the SM Higgs doublet. These doublets have the same quantum numbers and are consequently indistinguishable (at least at this step). Owing to this indistinguishability we can perform the rotation described above without any physical consequences (it is in fact a change of basis). It means that the model is invariant under a global $SO(2)$ transformation of the "bidoublet" $(\Phi_1\ \Phi_2)^{\mathrm{T}}$. However, it is very common to impose a discrete symmetry on the Higgs doublets $(\Phi_1 \to \Phi_1,\ \Phi_2 \to -\Phi_2)$ [9] or a global $U(1)$ symmetry $\left(\Phi_1 \to \Phi_1,\ \Phi_2 \to e^{i\varphi}\Phi_2\right)$ [6] to prevent dangerous flavor changing neutral currents or to study the Higgs sector of some new physics scenarios such as the minimal supersymmetric standard model. In that case, we are introducing a distinguibility between the doublets, because they acquire very different couplings to the fermions (models type I and II). Of course, we could have defined the symmetry in the opposite way, $(\Phi_1 \to -\Phi_1,\ \Phi_2 \to \Phi_2)$, but once we have chosen one of them, we cannot interchange $\Phi_1 \leftrightarrow \Phi_2$ anymore without changing the physical content. Such a fact breaks explicitly the $SO(2)$ symmetry of the "bidoublet". On the other hand, it is precisely this symmetry what allows us to absorb the $\tan\beta$ parameter, and since models type I and II do not have that symmetry, we are not able to absorb it properly.

There is another interesting way to see why the rotation can be carried out when C-invariance is imposed while it cannot be applied in the case of imposing a discrete symmetry. C-invariance (in a certain basis) requires the vanishing of the parameters $\mu_4^2, \lambda_8, \lambda_9, \lambda_{10}$; however, (10) shows that such parameters only transform among themselves i.e. the subset $\mu_4^2, \lambda_8, \lambda_9, \lambda_{10}$ is written in terms of the subset $\widetilde{\mu}_4^2, \widetilde{\lambda}_8, \widetilde{\lambda}_9, \widetilde{\lambda}_{10}$. Such transformations also show that if the subset in a certain basis vanishes, the corresponding subset in any other basis vanishes as well. By contrast, the discrete symmetry requires the vanishing of the subset $\mu_3^2, \mu_4^2, \lambda_6, \lambda_7, \lambda_8, \lambda_9$, but the elements of this subset do not transform among themselves as (10) shows, from which we see that this symmetry is not (in general) preserved by the rotation.

Finally, when we consider a phase in a VEV and make a complex rotation that eliminates such a VEV, the change of basis translates the source of CP violation from the VEV to the parameters of the potential. It means that the original spontaneous violation of CP has become an explicit violation of such a symmetry when the basis is changed.

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